CLASS XI – ANNUAL EXAMINATION

MATHEMATICS

(Maximum Marks: 100)

(Time allowed: Three hours)

*(Candidates are allowed additional 15 minutes for **only** reading the paper. They must NOT start writing during this time.)*

This Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C***

***Section A:** Internal choice has been provided in three questions of four marks each and two questions of six marks each.*

***Section B:** Internal choice has been provided in two questions of four marks each.*

***Section C:** Internal choice has been provided in two questions of four marks each.*

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A (80 Marks)

Question 1 **[10 × 2]**

(i) If $A = \{3n+5 : n \in N \text{ and } n \leq 6\}$ then represent Set A in the roster form.

(ii) If $A = \{x, y, z\}, B = \{a, x\}, C = \{a, 1, x\}$, then find $(A \times B) \cap (A \times C)$.

(iii) Prove that $\cot A + \tan(\pi + A) + \tan\left(\frac{\pi}{2} + A\right) + \tan(2\pi - A) = 0$

(iv) In Δ ABC, a = 2, b = 3, c = 4, then find cos A.

(v) Find the conjugate of $\frac{1}{3-4i}$

(vi) Given that α and β are the roots of the quadratic equation $px^2 + qx + 1 = 0$, find the value of $\alpha^3\beta^2 + \alpha^2\beta^3$.

(vii) In how many ways can the letters of the word 'MOTHER' be arranged to form words beginning with 'T'?

(viii) Find the derivative of $x^3 \cdot \sin x$ with respect to x.

(ix) Evaluate:

$$\lim_{x \to 0}\left[\frac{4\sin x - 9x\cos x}{3x^2 - 5\tan x}\right]$$

(x) From a pack of 52 well-shuffled cards, what is the probability of choosing a card that is either an Honour card or a Black card?

This Paper consists of 5 printed pages and one blank page.

Question 2 **[4]**

Find the domain and range of $f(x) = \frac{x}{2-3x}$

Question 3 **[4]**

(a) Prove that $1 + \frac{\cos 2\theta + \cos 6\theta}{\cos 4\theta} = \frac{\sin 3\theta}{\sin \theta}$.

OR

(b) Solve: $tan^2 A - (1+\sqrt{3})tan\ A + \sqrt{3} = 0, 0 \le A \le \frac{\pi}{2}$

Question 4 **[4]**

Using mathematical induction prove that :

$$3^{2n+2} - 8n - 9 \text{ is divisible by 64 for all } n \in N$$

Question 5 **[4]**

Illustrate in a complex plane the set of points Z satisfying

$$|Z + i - 2| \le 2 \text{ where } Z = x + iy$$

Question 6 **[4]**

(a) How many different arrangements can be made from the letters of the word SIGNIFICANTLY in such a way that:

(i) all the vowels are together.

(ii) all the vowels are not together.

OR

(b) A team of 5 members is to be formed from 5 men and 6 women. In how many ways can this be done if the team should consist of:

(i) at least 3 women.

(ii) at most 3 women.

Question 7 **[4]**

Find the fourth term from the end in the expansion of $\left[\frac{x^3}{2} - \frac{2}{x^2}\right]^9$

Question 8 **[4]**

Find the equation of the line passing through the point of intersection of the lines $2x + y - 3 = 0$ and $3x - 2y + 4 = 0$ and parallel to the line $4x + y = 7$.

Question 9 **[4]**

(a) Find the equations of the two tangents to the circle $x^2 + y^2 - 4x + 6y = 12$ which are perpendicular to the line $3x + 4y = 2$.

OR

(b) Find the equation(s) of a circle passing through the points (2, 3), (−1, 1) and whose centre is lying on the line $x - 3y = 11$.

Question 10 **[4]**

Using first Principle, find the derivative of: $2x^2 + 3x$.

Question 11 **[6]**

In any Δ ABC, prove that $\frac{b^2 - c^2}{a^2}$ sin 2A + $\frac{c^2 - a^2}{b^2}$ sin 2B + $\frac{a^2 - b^2}{c^2}$ sin 2C = 0

Question 12 **[6]**

(a) Find the range of x of the inequality $\frac{x^2 - 2x + 5}{3x^2 - 2x - 5} \geq \frac{1}{2}$

OR

(b) Find the value of p, such that the difference of the roots of the equation $x^2 - px + 10 = 0$ is 3. If the roots of $x^2 - px + 10 = 0$ are α, β find the quadratic equation whose roots are $(\alpha + \beta)^2$ and $\alpha^2\beta - \alpha\beta^2$.

Question 13 **[6]**

(a) An Arithmetic Progression has the first term as 2 and the fifth term as 30. A Geometric Progression has a common ratio of −0.5. The sum of the first two terms of the Geometric Progression is the same as the second term of the Arithmetic Progression. Find the first term of the Geometric Progression.

OR

(b) Find the sum of the series 3 + 5 + 11 + 29 + ... to n terms.

Question 14 **[6]**

Find the mean and standard deviation of the following frequency distribution:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of Students	4	16	20	10	7	3

SECTION B (20 Marks)

Question 15 **[3×2]**

(a) Write the converse and inverse of the statement:

$$\text{If } \Delta\,ABC \cong \Delta\,XYZ, \text{ then } \Delta\,ABC \sim \Delta\,XYZ$$

(b) Find the equation of the hyperbola with eccentricity $\frac{5}{3}$ and foci $(\pm 5, 0)$

(c) Show that the given statement is true by the method of contradiction:

$$\text{If } x \text{ is a real number such that } x^3 + 8x = 0, \text{thus } x = 0$$

Question 16 **[4]**

(a) Prove that the line $5x + 12y = 9$ touches the hyperbola $x^2 - 9y^2 = 9$. Also find the point of contact.

OR

(b) Find the equation of the ellipse whose foci are (-3, 5) and (5, 5) and whose major axis is 10.

Question 17 **[4]**

(a) Find the ratio in which yz plane divides the line joining A (2, 4, 5) and B (3, 5, –4). Also, find the point of intersection.

OR

(b) Find the fourth vertex of the parallelogram whose consecutive vertices are (2, 4, –1), (3, 6, –1) and (4, 5, 1). Hence or otherwise find the length of the diagonal.

Question 18 **[6]**

Find the equation of the parabola whose vertex is the point (3, 4) and focus is the point (1, 2).

SECTION C (20 Marks)

Question 19

(a) The mean mark of all students of Classes XI A and XI B in an examination is 45. The mean mark of Classes XI A and XI B was 30 and 50 respectively. Determine the ratio of the students of Classes XI A and XI B. **[2]**

(b) (i) Compute Q1, Q2 and Q3 for the following data of marks obtained by 50 students in a test: **[4]**

Marks:	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of students:	5	9	14	13	9

OR

(ii) Calculate the mode of the following data:

Class Interval	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency	4	6	14	10	6

Question 20

(a) Find Cov (x, y) if $\sum x_i = 15$, $\sum y_i = 40$, $\sum x_i\, y_i = 150$ and $n = 5$ **[2]**

(b) (i) Find the Karl Pearson's Coefficient of Correlation between the values of x and y given below: **[4]**

x	15	12	8	10	9	6	5	12	6	3
y	13	18	12	14	15	10	8	20	12	6

OR

(ii) Find Spearman's rank correlation coefficient for the data given below:

X	15	14	13	16	22	15	19	13	23
Y	21	13	12	20	18	14	15	17	16

Question 21 **[4]**

The Price Relative and Weights of a set of Commodities are as follows:

Commodity	A	B	C	D
Price Relative	125	120	127	119
Weight	x	$2x$	y	$y + 3$

Given that the sum of the weight is 40 and that the index number is 122, calculate numerical values of x and y.

Question 22 **[4]**

Obtain the three year moving average for the following series of observations:

Year	1995	1996	1997	1998	1999	2000	2001	2002
Annual sales (₹ in thousands)	3·6	4·3	4·3	3·4	4·4	5·4	3·4	2·4

and show it on a graph paper.